WEEKLY TEST RANKER'S BATCH-01 TEST - 05 Balliwala SOLUTION Date 13-10-2019

[PHYSICS]

1. $\eta = 1 - \frac{T_L}{T_H}$

2.

3. C_v for hydrogen $= \frac{5R}{2}$, C_0 for helium $= \frac{3R}{2}$

C_v for water vapour $= \frac{6R}{2}$

$$\therefore \quad [C_v]_{mix} = \frac{4 \times \frac{5R}{2} + 2 \times \frac{3R}{2} + 1 \times 3R}{4+2+1} = \frac{16R}{7}$$

$$\therefore \quad C_p = C_v + R = \frac{16R}{7} + R = \frac{23R}{7}$$

4. As pressure is varrying linearly with volume, work done
dW = area under PV curve

$$= P_1(V_F - V_1) + \frac{1}{2}(P_F - P_I)(V_F - V_I)$$

$$= \frac{1}{2}(P_F + P_I)(V_F \_ V_I)$$

$$dU = \mu C_v dT = \mu \frac{R}{\gamma - 1} dT = \frac{P_F V_F - P_I V_I}{\gamma - 1}$$

$$\therefore \quad dU = \frac{10^5 (8 \times 0.5) - 4 \times 0.2)}{(5/2) - 1} = 4.8 \times 10^5 \text{ J}$$

5. $PV\gamma$ = constant
$P\gamma V^{\gamma-1}\gamma V + \Delta PV^{\gamma} = 0$

$$\therefore \quad \frac{\Delta P}{P} = -\gamma \frac{\Delta V}{V}$$

6.

7. In cylinder A, heat is supplied at constant pressure while in cylinder B heat is supplied at constant volume.

$(\Delta Q)_A = nC_p(\Delta T)_A$

and $(\Delta Q)_B = nC_v(\Delta T)_B$

Given that :

$(\Delta Q)_A = (\Delta Q)_B$

$$\therefore \quad (\Delta T)_B = \frac{C_p}{C_v}(\Delta T)_A = (1.4)(30) = 42 \text{ K}$$

11. Since, P–V graph of the process is a stright line and two points $(V_0, 2P_0)$ and $(2V_0, P_0)$ are known, therefore its equation will be,

$$(P - P_0) = \frac{(2P_0 - P_0)}{(V_0 - 2V_0)}(V - 2V_0) = \frac{P_0}{V_0}(2V_0 - V)$$

$$\therefore \quad P = 3P_0 = \frac{P_0 V}{V_0}$$

According to equation for ideal gas,

$$T = \frac{PV}{nR}$$

$$= \left(3P_0 - \frac{P_0 V}{V_0}\right)\frac{V}{nR}$$

$$= \frac{3P_0 V_0 V - P_0 V^2}{nRV_0} \qquad ......(i)$$

For T to be maximum, $\frac{dT}{dV} = 0$

$$3\,3P_0 V_3 - 2P_0 V = 0$$

or $V = \frac{3V_0}{2}$ ......(ii)

Putting this value in equation(i), we get,

$$T_{max} = \frac{3P_0 V_0\left(\frac{3V_0}{2}\right) - P_0\left(\frac{9}{4}V_0^2\right)}{nRV_0} = \frac{9P_0 v_0}{4nR}$$

12. In the curves 1-2 and 3-4, we find that the pressure is directly proportional to temperature. So, the volume remains unchanged, i.e., gas does not work.

The work done during the isobaric processes 2-3 and 1-4 are as follows

$W_{2-3} = P_2(V_3 - V_2)$

Total work done $= P_2(V_3 - V_2) + P_1({}_1V - V_4)$

$W_T = P_2V_3 - P_2V_2 + P_2V_2 - P_1V_4$

Three moles has been given, so

$PV = nRT = 3RT$

$\therefore$ $W_T = 3RT_3 - 3RT_2 + 3RT_1 - 3RT_4$

$= 3R[T_1 + T_3 - T_2 - T_4]$

$= 3R[400 + 2400 - 800 - 1200]$

$= 3R \times 800 = 20 \times 10^3$ J = 20 kJ

13. According to first law of thermodynamics :

$\Delta Q = \Delta U + \Delta W$

For the process abc, $80 = \Delta U + 60$

or $\Delta U = 20$ cal

Since, ΔU is independent of path the internal energy change is same for both the paths abc and adc.

For the process adc, $\Delta Q = \Delta U + \Delta W$

$\therefore$ $\Delta W = \Delta Q - \Delta U = 30 - 20 = 10$ cal

14.

15. In adiabatic expansion we know that dQ = 0

dQ = dU + PdV or 0 = dU + PdV

PdV = – dU

Thus, work done decrease internal energy which is function of temperature. Hence, temperaure also decreases.

16. For mixture of gases, let specific heat be C_1

$$C_v = \frac{n_1(C_v)_1 + n_2(C_v)_2}{n_1 + n_2}$$

where $(C_v)_1 = \frac{5R}{2}, (C_v)_2 = \frac{3R}{2}$

$\therefore \quad C_v = \frac{2 \times \frac{5R}{2} + 8 \times \frac{3R}{2}}{2+8} = \frac{17R}{10} = 1.7R$

17. Two chambers are kept separate at (P, 5V) and (10P, V). When the piston is allowed to move, the gases are kept separated but the pressure has to be equal
Let the equal new pressure on the both sides of piston in its equilibrium position be P'.
As the process is isothermal, hence
for LHS portion: $P(5V) = P'(5V - x)$ .....(i)
and for RHS portion : $10P(V) = P'(V + x)$ ....(ii)
solving equations (i) and (ii) we get;
$x = 3V$
$\therefore$ New volume of LHS portion = 2V and new volume of RHS portion = 4V

18. PV^{γ} = constant for an adiabatic process
Given : $P^3 \times V^4$ = constant

or $\quad PV^{4/3}$ = constant $\quad \left(\text{as } P^3 \propto \frac{1}{V^4}\right)$

or $\quad \gamma = \frac{4}{3} = 1.33$

19.

20. From first law of thermodynamics,
$Q = \Delta U + W$
For path iaf, $50 = \Delta U + 20$
$\therefore$ $\Delta U = U_f - U_i = 30$ cal
For path ibf, $Q = \Delta U + W$
or $\quad W = Q - \Delta U$
$= 36 - 30 = 6$ cal

21. In cyclic process, the amount of heat given to system is equal to the net work done by the system.
Hence, correct answer is (b)

22. Work done in complete cycle
= Area under P - V graph
$= P_0V_0$
Heat given to the gas in going from A to B

$nC_v g\Delta T = n.\frac{3}{2}R\Delta T$

Heat given to the gas in going from B to C

$= nC_v\Delta T = n\left(\frac{5}{2}R\right)\Delta T$

$= \frac{5}{2}(2P_0)\Delta T = 5P_0V_0$

Heat is rejected in going from C to D and then D to A.

Effeciency, $\eta = \frac{\text{Work done by the gas}}{\text{Heat given to the gas}} \times 100$

$= \frac{P_0V_0 \times 100}{\frac{3}{2}P_0V_0 + 5P_0V_0} = \frac{2 \times 100}{13} = 15.4\%$

23. $\frac{K_{Fe}A(100-\theta)}{d} = \frac{K_{Ag}(\theta - 0)}{d}$

$\frac{K_{Fe}}{K_{Ag}} = \frac{\theta}{100-\theta}$ or $\frac{1}{11} = \frac{\theta}{100-\theta}$

$\therefore \quad \theta = \frac{100}{12} = 8.3^oC$

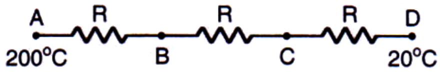
24. $\frac{dQ}{dt} = \frac{dQ_1}{dt} + \frac{dQ_2}{dt}$

$\frac{K(A_1 + A_2)(\theta_1 - \theta_2)}{d} = \frac{K_1A_1(\theta_1 - \theta_2)}{d} + \frac{K_2A_2(\theta_1 - \theta_2)}{d}$

$\therefore \quad K = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$

25.

26. Equivalent electrical circuit, will be as shown in figure.



Temperature difference between A and D is 180ºC, which is equally distributed in all the rods. Therefore, temperature difference between A and B will be 60ºC, or temperature of B should be 140ºC

27.

28. Rate of flow of heat $\frac{dQ}{dt}$ or H is equal throughout the rod. Temperature difference is given by :

T.D. = (H) (Thermal Resistance)
or R.D $\propto$ Thermal Resistance R

Where, $R = \frac{1}{KA}$ or $R \propto \frac{1}{A}$

Area across CD is less. Therefore, T.D. cross CD will be more.

29. Here, $K_1 = K_2$, $l_1 = l_2 = 1m$,
$A_1 = 2A$, $A_2 = A$
$T_1 = 100^oC$, $T_2 = 70^oC$
$\therefore$ Temperaure at C be T, then

$\frac{\Delta Q}{\Delta t} = \frac{K2A(100 - T)}{1} = \frac{KA(T - 70)}{1}$

or T = 90º

30. We know that : $\frac{dQ}{dt} = KA\frac{d\theta}{dx}$

In steady state flow of heat,

$d\theta = \frac{dQ}{dt}.\frac{1}{KA}dx$

or $\theta_H - \theta = K'x$
or $\theta = \theta_H - K'x$
Equation, $\theta = \theta_H - K'x$ represents a straight line.

31.

32.

33. For parallel combination of two rods of equal length and equal area of cross-section :

$K = \frac{K_1 + K_2}{2} = \frac{K_1 + \frac{4K_1}{3}}{2} = \frac{7K_1}{6}$

Hence, $\frac{K}{K_1} = \frac{7}{6}$

34. When a body cools by radiation, according to Stefan's law,

$$\frac{dT}{dt}\frac{eA\sigma}{mc}\left(T^4 - T_0^4\right)$$

Here, m, c, e, T and T_0 are same for all bodies; so

$$\frac{dT}{dt} \propto \text{area A}$$

Now, as for a given mass, area of the sphere is minimum, hence it will have the lowest rate of cooling.

35. Radius of small sphere = r and thicknes = t
Radius of bigger sphere = 2r and thickness = t/4

Mass of ice melted $= \frac{4}{3}\pi r^3 p$

For bigger sphere : $\frac{K_1 4\pi(2r^2)\times 100}{t/4} = \frac{\frac{4}{3}\pi(2r^2)\rho L}{25\times 60}$

For smaller sphere : $\frac{K_2 4\pi r^2 \times 100}{t} = \frac{\frac{4}{3}\pi r^2\rho L}{16\times 60}$

$$\frac{K_1}{K_2} = \frac{8}{25}$$

36. According to Wein's law
$\lambda_m T$ = constant
or $(\lambda_m)_1 T_1 = (\lambda_m)_2 T_2$
or $11 \times 10^{-5} \times T_1 = (5.5 \times 10^{-5}) \times T_2$

$\therefore \quad T_1 = \frac{1}{2}T_2$

37. $P \propto \frac{T^4}{d^2}$

When T and d are doubled, then power received by the surface

$$P' \propto \frac{(2T)^4}{(2d)^2} \propto \frac{2^4 T^4}{2^2 d^2} \propto \frac{4T^4}{d^2}$$

Hence, P' = 4P

38.
39.
40.
41.
12. According to Newton's law of cooling, rate of loss of heat $\propto (T - T_0)$ where T is the average temperature in the given time internal. Hence,

43. $\frac{Q}{t} \propto K \quad \text{or } Kt = \text{constant}$

$\therefore \quad \frac{K_1}{K_2} = \frac{t_2}{t_1} = \frac{40}{20} = \frac{2}{1}$

44. For slabs in series, we have,

$R_{eq.} = R_1 + R_2$

i.e., $\frac{5x}{K_{eq.}A} = \frac{4x}{2KA} + \frac{x}{KA}$

Now, is steady state, rate of heat transfer through the slab

$$\frac{K_{eq.}A\left(T_2 - T_1\right)}{5x} = \left(\frac{5\left(T_2 - T_1\right)K}{x}\right)f$$

Putting the value of K_{eq}, we get;

$$f = \frac{1}{3}$$

45. $\frac{Q}{At} = \frac{K\left(\theta_1 - \theta_2\right)}{d} = \text{constant}$

$\therefore$ $K_A\left(\frac{\theta_1 - \theta}{d}\right) = K_B\left(\frac{\theta - \theta_2}{d}\right)$

$\frac{K_A}{K_B} = \frac{\theta - \theta_2}{\theta_1 - \theta}$ or $3 = \frac{\theta - \theta_2}{\theta_1 - \theta}$

or $3\theta_1 - \theta_2 = 4\theta$ .....(i)

Given $\theta_1 - \theta_2 = 20^oC$ .....(ii)

Solving eqns. (i) & (ii), we have $\theta - \theta_2 = 15^oC$

$\therefore$ q – q = $\theta_1 - \theta_2 + \theta_2 - \theta$

$= (\theta_1 - \theta_2) - (\theta - \theta_2)$

$= 20^oC - 15^oC = 5^oC$